Generalized action invariants for drift waves-zonal flow systems

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Abstract

Generalized action invariants are identified for various models of drift wave turbulence in the presence of the mean shear flow. It is shown that the wave kinetic equation describing the interaction of the small scale turbulence and large scale shear flow can be naturally writen in terms of these invariants. Unlike the wave energy, which is conserved as a sum of small- and large- scale components, the generalized action invariant is shown to correspond to a quantity which is conserved for the small scale component alone. This invariant can be used to construct canonical variables leading to a different definition of the wave action (as compared to the case without shear flow). It is suggested that these new canonical action variables form a natu-

ral basis for the description of the drift wave turbulence with a mean shear flow.

The dynamics of the small scale turbulence in the presence of a mean shear flow is a problem of a great interest for plasmas and geostrophic fluids. It is believed that the nonlinear energy transfer from small to large length scale component (inverse cascade [1]) is a cause of a spontaneous generation and sustainment of coherent large structures, e.g. zonal flows in atmospheres, ocean and plasmas [2]. In the few past years it has been suggested [3, 4, 5, 6, 7, 8, 9, 10] that the large scale flow band structures (zonal flows) play an important role in regulating and suppressing the anomalous transport in magnetic confinement systems.

In the simplest form, the generation of plasma flow by turbulence can be described by the energy conservation relation (Poynting theorem) averaged over small scale fluctuations [9]. A generalization of this approach is a WKB type wave kinetic equation for the quanta density of small scale fluctuations that is conserved along the rays. This method was originally proposed in Ref. 11 to describe the interaction of high frequency plasmons (Langmuir waves) with low frequency ion sound perturbations. It is widely used also in general fluid dynamics [12].

In studies of drift wave dynamics, it has been naturally assumed [13, 14] that the relevant quantity that is conserved in the presence of slow variations is the drift-wave action density. It is well known [15], that the standard wave action variables C_k associated with the number of wave quanta n_k, $n_k = |C_k|^2 = E_k/\omega_k$, where E_k is the wave energy, and ω_k is the wave frequency, is a basis for Hamiltonian form of the wave-wave interaction equations. It has been noted in Refs. 16,17 that the normal variables used to describe self-interaction between small scale fluctuations without the shear flow are modified by the flow and may not be suitable for a system with a mean flow. Thus, in the presence of a shear flow a new form of canonical variables and associated action invariant have to be identified. On other hand, it has been pointed[18] that the conserved action-like quantity (pseudo-action) is different from the usual definition of the wave action defined as the ratio of the wave energy to the wave frequency. The latter definition is also fails when there are no oscillating eigenmodes such as in ideal fluid, so that an alternative definition of the action-like integral is required[19].

It is important to realize that the natural form of the three-wave interaction equations for the drift-waves does not have Hamiltonian structure [20].These equations can be transformed, however, to a Hamiltonian form via an asymptotic variable transformation. Such a tranformation yielding a Hamiltonian form for the drift and Rossby waves has been found in Refs. 20,21. There are several possible forms for such a transformation. In Refs. 17,20,21 it is based on the conserved energy integral that leads to the standard definition of the wave action. For drift-wave+zonal flow systems small scales are modulated by larger scale shear flows so that energy in the small scale component is not conserved. Thus, the canonical Hamiltonian variables constructed from energy conservation are not suitable for description of the drift waves in the presence of a mean flow.

In this paper, we derive the WKB type wave kinetic equation that describes the conservation (along the rays) of an action like invariant of the drift wave turbulence with slowly varying parameters due to the mean sheared flow. We demonstrate that the relevant action-like integral corresponds to the quantity conserved for the small scale component alone. We show that the structure of the action integral is determined by the structure of the matrix element describing the interaction of the small scale and large scale component. We discuss how the canonical variables corresponding to such a pseudo-action invariant can be constructed.

The scale separation between the small scale turbulence and the large scale motions is an essential property of drift-wave+zonal flow systems that is commonly used [12, 16, 17, 18, 19, 22, 23] to simplify the analysis. Though, the scale separation is often observed experimentally and in computer simulation, it may be less pronounced in other cases[24]. In our present paper, we substantially rely on the multiscale expansion, so our results are valid, strictly speaking, only in the case when there is such a scale separation. More general approach avoiding the scale separation assumption, namely the renormalization group, is possible [25], but it is beyond the scope of the present paper.

We consider a generic case of the drift wave equation in the form

$$\frac{\partial \phi_k}{\partial t} + i\omega_k \phi_k + \int d^2p L_{p,k-p}\phi_p\phi_{k-p} = 0, \tag{1}$$

where $\omega_k = \omega(k)$ is the frequency of the linear mode with a wavector k, and may include an imaginary part corresponding to the wave grow and decay.

In the spirit of the scale separation we represent the field into the large-scale $\phi_k^<$ and small-scale $\phi_k^>$ components; $\phi_k^< = 0$ outside a shell $\mid \mathbf{k} \mid < \varepsilon \ll 1$, $\phi_k^> = 0$ for $\mid \mathbf{k} \mid < \varepsilon$.

Assuming that the self-interaction of small-scale fields is small compared to the interaction with the mean flow[17] we write from (1) the following equation for the small-scale fluctuations

$$\frac{\partial \phi_k^>}{\partial t} + i\omega_k \phi_k^> + \int d^2p L_{p,k-p}\phi_p^<\phi_{k-p}^> = 0. \tag{2}$$

To derive the equation for the evolution of the wave spectrum we multiply equation (2) by $\phi_{k'}^>$ and then add it with a similar equation obtained by reversing k and $k^{'}$, yielding

$$\frac{\partial}{\partial t}\left(\phi_k^>\phi_{k'}^>\right) + i\left(\omega_k + \omega_{k'}\right)\ \phi_k^>\phi_{k'}^> + \phi_{k'}^> \int d^2p L_{p,k-p}\phi_p^<\phi_{k-p}^> + \phi_k^> \int d^2p L_{p,k'-p}\phi_p^<\phi_{k'-p}^> = 0. \tag{3}$$

The small-scale turbulence is described by the spectral function (Wigner function) $I_k(\mathbf{x}, t)$, defined as follows

$$\int d^2q \left\langle \phi_{-k+q}^> \phi_k^> \right\rangle \exp(i\mathbf{q}\cdot\mathbf{x}) = I_k(\mathbf{x}, t). \tag{4}$$

The slow time and spatial dependence in $I_k(\mathbf{x}, t)$ corresponds to modulations with a "slow" wavevector, $\mathbf{q} \ll \mathbf{k}$. Angle brackets in (4) stand for ensemble average, which is equivalent to a time average with appropriate ergodic assumptions.

The equation for $I_k(\mathbf{x}, t)$ is derived from (3) by averaging it over fast scales and by taking the Fourier transform over the slow variable $\mathbf{x}$. Setting $\mathbf{k}^{'} = -\mathbf{k} + \mathbf{q}$ and applying the operator $\int d^2q \exp(i\mathbf{q} \cdot \mathbf{x})$ we obtain

$$\frac{\partial}{\partial t} I_k(\mathbf{x}, t) + i \int d^2q \exp(i\mathbf{q} \cdot \mathbf{x}) \left(\omega_k + \omega_{-k+q}\right) \left\langle \phi_k^{>} \phi_{-k+q}^{>} \right\rangle + S_1 + S_2 = 0, \quad (5)$$

$$S_1 = \int \int d^2p d^2q \exp(i\mathbf{q} \cdot \mathbf{x}) \left\langle \phi_{-k+q}^{>} \phi_{k-p}^{>} \right\rangle L_{p,k-p} \phi_p^{<}, \quad (6)$$

$$S_2 = \int \int d^2p d^2q \exp(i\mathbf{q} \cdot \mathbf{x}) \left\langle \phi_{-k+q-p}^{>} \phi_k^{>} \right\rangle L_{p,-k+q-p} \phi_p^{<}. \quad (7)$$

The second term in (3) gives

$$i \int d^2q \exp(i\mathbf{q} \cdot \mathbf{x}) \left(\omega_k + \omega_{-k+q}\right) \left\langle \phi_k^{>} \phi_{-k+q}^{>} \right\rangle = \frac{\partial \omega_k}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{x}} I_k(\mathbf{x}, t) - 2\gamma_k I_k, \quad (8)$$

where γ_k is the linear growth rate, and only the real part of the frequency is presumed for ω_k on the right hand side of this equation.

The ensemble average in S_1 can be transformed by using the inverse of (4)

$$\left\langle \phi_{-k+q}^{>} \phi_{k-p}^{>} \right\rangle = \left\langle \phi_{k-p}^{>} \phi_{-(k-p)+q-p}^{>} \right\rangle = \int d^2x^{'} I_{k-p}(x^{'}) \exp(-i(\mathbf{q} - \mathbf{p}) \cdot \mathbf{x}^{'}). \quad (9)$$

By using (9) and expanding in $\mathbf{p} \ll \mathbf{k}$ the expression for S_1 is transformed to

$$S_1 = \int d^2p \exp(i\mathbf{p}\cdot\mathbf{x}) L_{p,k-p} \left(I_k(\mathbf{x}) - \mathbf{p} \cdot \frac{\partial I_k(\mathbf{x})}{\partial \mathbf{k}} \right) \phi_p^< \; . \tag{10}$$

Similarly, by using the identity analogous to (9) and expanding the interaction coefficient $L_{p,k-p}$ in $\mathbf{p} \ll \mathbf{k}$, we transform S_2 to the form

$$\begin{aligned} S_2 &= \int\int d^2p d^2q \exp(i\mathbf{q}\cdot\mathbf{x}) \left(L_{p,-k} + (\mathbf{q}-\mathbf{p}) \cdot \frac{\partial L_{p,-k}}{\partial(-\mathbf{k})} \right) \phi_p^< \\ &\quad \times \int d^2x' \;\; \exp(-i(\mathbf{q}-\mathbf{p})\cdot\mathbf{x}') I_k(x') \\ &= I_k(x) \int d^2p \exp(i\mathbf{p}\cdot\mathbf{x}) L_{p,-k} \phi_p^< - i \int d^2p \exp(i\mathbf{p}\cdot\mathbf{x}) \frac{\partial L_{p,-k}}{\partial(-\mathbf{k})} \cdot \frac{\partial I_k}{\partial \mathbf{x}} \phi_p^< \end{aligned} \tag{11}$$

Equations (5-11) define a particular form of the transport equation for $I_k(\mathbf{x}, t)$ for a given interaction coefficient $L_{k,k'}$.

In this paper, we consider two different models for drift waves in a magnetized plasma: the standard Hasegawa-Mima equation and a slab-like model for drift waves in a sheared magnetic field. The latter is similar to the standard Hasegawa-Mima equation with a modified plasma response to the slow modulations of the electrostatic potential. Such slow modes correspond to $k_{\parallel} \to 0$, so that the slow part of the potential does not follow Boltzmann distribution. [Note that zonal flows[10] ($m = n = 0$) are such slow modes with $k_{\parallel} = 0$.] As a result, the convective term appears in the lowest order, contrary to the case of the Hasegawa-Mima equation where such term is due to the polarization drift. Appropriate equation for the drift wave dynamics in presence of a mean flow (neglecting the self-interaction) has the form [13]

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_0 \cdot \nabla\right) \frac{e\widetilde{\phi}}{T_e} + \mathbf{V}_* \cdot \nabla \frac{e\widetilde{\phi}}{T_e} - \rho_s^2 \left(\frac{\partial}{\partial t} + \mathbf{V}_0 \cdot \nabla\right) \nabla_\perp^2 \frac{e\widetilde{\phi}}{T_e} = 0. \tag{12}$$

where $\mathbf{V}_0 = c\mathbf{b}\times\nabla\overline{\phi}/B_0$ is the mean flow velocity. This equation can be written in the form (2) with $\omega_k = \mathbf{k}\cdot\mathbf{V}_*/(1+k^2\rho_s^2)$ and

$$L_{k_1,k_2} = -\frac{c}{B_0}\frac{\mathbf{b}\cdot\mathbf{k}_1\times\mathbf{k}_2}{1+(\mathbf{k}_1+\mathbf{k}_2)^2\rho_s^2}\left(1+k_2^2\rho_s^2\right). \tag{13}$$

From (5-11) and (13) we obtain

$$\frac{\partial}{\partial t} I_k(\mathbf{x},t) + \frac{\partial}{\partial \mathbf{k}}\left(\omega_k + \mathbf{k}\cdot\mathbf{V}_0\right)\cdot\frac{\partial I_k}{\partial \mathbf{x}} - \frac{\partial}{\partial \mathbf{x}}\left(\frac{\mathbf{k}\cdot\mathbf{V}_0}{(1+k^2\rho_s^2)^2}\right)\cdot\frac{\partial}{\partial \mathbf{k}} I_k (1+k^2\rho^2)^2 = 0. \tag{14}$$

This equation can be written in the form of a conservation law for the invariant $N_k = I_k(1+k^2\rho^2)^2$,

$$\frac{\partial}{\partial t} N_k(\mathbf{x},t) + \frac{\partial}{\partial \mathbf{k}}\left(\omega_k + \mathbf{k}\cdot\mathbf{V}_0\right)\cdot\frac{\partial N_k}{\partial \mathbf{x}} - \frac{\partial}{\partial \mathbf{x}}\left(\mathbf{k}\cdot\mathbf{V}_0\right)\cdot\frac{\partial}{\partial \mathbf{k}} N_k = 0. \tag{15}$$

By direct evaluation from (12), it can be easily shown that the quantity

$$N = \int d^2k \left(\widetilde{\phi}^2 + 2\rho_s^2(\nabla_\perp\widetilde{\phi})^2 + \rho_s^4(\nabla_\perp^2\widetilde{\phi})^2\right), \tag{16}$$

corresponding to N_k in (17), is conserved as an integral over the small-scale part of the spectrum. In (16) $\widetilde{\phi}$ is the normalized potential of the small scale fluctuations. This property distinguishes N_k from any other combination of the energy and enstrophy which are conserved only as a sum of contributions from the small and long scale parts of the spectrum[22].

A different expression for the action-like invariant is obtained for the standard Hasegawa-Mima (H.M.) model with a mean flow

$$\frac{\partial}{\partial t}\left(\frac{e\widetilde{\phi}}{T_e} - \rho_s^2\nabla_\perp^2\frac{e\widetilde{\phi}}{T_e}\right) + \mathbf{V}_*\cdot\nabla\frac{e\widetilde{\phi}}{T_e} - \rho_s^2(\mathbf{V}_0\cdot\nabla)\nabla_\perp^2\frac{e\widetilde{\phi}}{T_e} = 0. \tag{17}$$

The appropriate interaction coefficient is

$$L_{k_1,k_2} = -\frac{c}{2B_0}\rho_s^2\frac{\mathbf{b}\cdot\mathbf{k}_1\times\mathbf{k}_2}{1+(\mathbf{k}_1+\mathbf{k}_2)^2\rho_s^2}\left(k_2^2-k_1^2\right). \tag{18}$$

In this case, from (5-11) and (18) the transport equation for I_k takes the form

$$\begin{aligned}\frac{\partial}{\partial t}I_k + \frac{\partial}{\partial\mathbf{k}}\left(\omega_k + \frac{\mathbf{k}\cdot\mathbf{V}_0}{1+k^2\rho_s^2}k^2\rho_s^2\right)\cdot\frac{\partial}{\partial\mathbf{x}}I_k \\ -\frac{\partial}{\partial\mathbf{x}}\left(\frac{\mathbf{k}\cdot\mathbf{V}_0}{(1+k^2\rho_s^2)^2}\right)\cdot\frac{\partial}{\partial\mathbf{k}}k^2\rho_s^2(1+k^2\rho_s^2)I_k = 0.\end{aligned} \tag{19}$$

Obviously, this equation can be written in the form of the conservation law for the invariant $N_k = I_k k^2\rho_s^2(1+k^2\rho_s^2)$,[18, 22, 23]

$$\frac{\partial}{\partial t}N_k + \frac{\partial}{\partial\mathbf{k}}\left(\omega_k + \frac{\mathbf{k}\cdot\mathbf{V}_0}{1+k^2\rho_s^2}k^2\rho^2\right)\cdot\frac{\partial}{\partial\mathbf{x}}N_k - \frac{\partial}{\partial\mathbf{x}}\left(\frac{\mathbf{k}\cdot\mathbf{V}_0}{(1+k^2\rho_s^2)}k^2\rho_s^2\right)\cdot\frac{\partial}{\partial\mathbf{k}}N_k = 0. \tag{20}$$

Similarly to the previous case, the invariant N_k corresponds to the integral of (17) conserved for the small scale component alone

$$N = \int d^2k\ \rho_s^2\left((\nabla_\perp\widetilde{\phi})^2 + \rho_s^2(\nabla_\perp^2\widetilde{\phi})^2\right), \tag{21}$$

Note that both invariants (16) and (21) are different from standard definition of the wave action [13, 14]. The difference between two forms of the action-like invariant (Eq. (16) and (21)) is due to a different form of the coupling matrix (Eq. (13) and Eq.(18)) describing interaction of the small and large scale components.

The procedure that we have described above can also be used to derive the action-like invariant for the two-dimensional motion of an incompressible fluid. In the latter case, there are no oscillating modes so that the standard definition of the action as a ratio of the wave energy to wave frequency is not applicable. The 2-D Euler equation has a form

$$\partial \nabla_{\perp}^{2} \phi + \mathbf{V}_0 \cdot \nabla \nabla_{\perp}^{2} \phi = 0, \tag{22}$$

where $\mathbf{V}_0$ is the velocity due to the mean flow. This equation can be written in the form (1) with $\omega_k = 0$ and the interaction coefficient

$$L_{k_1,k_2} = -\frac{\mathbf{b} \cdot \mathbf{k}_1 \times \mathbf{k}_2}{(\mathbf{k}_1 + \mathbf{k}_2)^2} k_2^2. \tag{23}$$

Using equations (5-11) and (23) we obtain the wave kinetic equation

$$\frac{\partial}{\partial t} N_k(\mathbf{x}, t) + \frac{\partial}{\partial \mathbf{k}} (\mathbf{k} \cdot \mathbf{V}_0) \cdot \frac{\partial N_k}{\partial \mathbf{x}} - \frac{\partial}{\partial \mathbf{x}} (\mathbf{k} \cdot \mathbf{V}_0) \cdot \frac{\partial}{\partial \mathbf{k}} N_k = 0, \tag{24}$$

where the wave-action $N_k = k^4 I_k$ [19].

We summarize generalized wave action integrals for different models in the Table I. Note that the standard expression for the drift wave action defined as the ratio of the wave energy to the wave frequency is [13, 14]

$$n_k = |a_k|^2 = \frac{(1 + \rho_s^2 k_\perp^2)^2}{\omega_*} |\phi_k|^2 = \frac{E_k}{\omega_k}, \tag{25}$$

where $\omega_* = k_\theta V_*$. Expression (25) should be compared with the first two lines in the Table. It is interesting to note that generalized action invariant given by Eq. (16) coincides with the standard definition of the wave action (25) for the poloidally symmetric case when the poloidal wave vector k_θ is constant ($k_\theta = const$).

Next we consider the self-interaction between small scales in the presence of the shear flow and outline how the pseudo-action invariants can be used to construct the canonical variables for the latter case. For illustration, we consider the case of Hasegawa-Mima equation (17). We restore the self-interaction term given by W_{k,k_1,k_2}

$$\frac{\partial \phi_k}{\partial t} + i\omega_k \phi_k = \int d^2k_1 d^2k_2 W_{k,k_1,k_2} \delta(k - k_1 - k_2)\phi_{k_1}\phi_{k_2}, \tag{26}$$

$$W_{k,k_1,k_2} = -\frac{c}{2B_0}\rho_s^2 \frac{\mathbf{b}\cdot\mathbf{k}_1 \times \mathbf{k}_2}{1+\mathbf{k}^2\rho_s^2}\left(k_2^2 - k_1^2\right). \tag{27}$$

This natural form of the three-wave interaction does not have standard Hamiltonian structure. This is reflected in the interaction coefficients W_{k,k_1,k_2} which do not have the required symmetry properties [15]. The only symmetries in W_{k,k_1,k_2} are of the type $W^*_{-k,-k_1,-k_2} = W_{k,k_1,k_2} = W_{-k,k_1,k_2}$. Transformation of (26) to normal canonical variables a_k was given in Refs. 19,20 (see also Ref. 16). It has the form [17]

$$a_k = g_k \phi_k + \int d^2k_1 d^2k_2 G_{k,k_1,k_2} \delta(k - k_1 - k_2)\phi_{k_1}\phi_{k_2}. \tag{28}$$

In new variables the interaction coefficients V_{k,k_1,k_2} are

$$V_{k,k_1,k_2} = \frac{1}{3g_{k_1}g_{k_2}g_k}\left(|g_k|^2 W_{k,k_1,k_2} + |g_{k_1}|^2 W_{k1,k,k_2} + |g_{k_2}|^2 W_{k_2,k_1,k}\right) \tag{29}$$

These interaction coefficients V_{k,k_1,k_2} now have all symmetries required for Hamiltonian systems. The function g_k can be chosen in a variety of ways. The standard approach [17, 20, 21] is to chose g_k so that the energy in canonical variables takes the form $E = \int d^2k\omega_k a_k a_{-k}$. Comparing it with the energy integral $E = \int d^2k \left(\widetilde{\phi}^2 + \rho_s^2(\nabla_\perp\widetilde{\phi})^2)\right)$,we find [17] $g_k = (1 + \rho_s^2 k_\perp^2)/(k_y)^{1/2}$. This gives a standard expression for the wave action (25).

As discussed above, for the drift waves-zonal flow system the energy in the small scale component is not conserved, bur rather the total energy of drift waves + large scale zonal flows is constant. For this reason, the energy integral of the small scale component can not be used for introduction of canonical variables for self interaction of the small scale fluctuations. Contrary to the energy, the integrals N_k are conserved for small scale component. Choosing the function g_k such as that the invariants (16) or (21) are in the form $N_k = \int d^2k a_k a_{-k}$,we obtain N_k as canonical variables for drift waves in the presence of the mean shear flow. This automatically means that these invariants have a meaning of the generalized wave action invariant. Then, to account for the self-interactions in the presence of the background shear flow, the wave kinetic equation (Eq. (15) or (20)) should be modified with a source term J_k in the standard form[15]

$$J_k = 4\pi \int d^2k_1 d^2k_2 \times |V_{k,k_1,k_2}|^2 \left(N_{k_1}N_{k_2} - N_k N_{k_1} - N_k N_k\right) \delta(k - k_1 - k_2). \tag{30}$$

We have formulated a wave kinetic equation and determined a structure of an appropriate adiabatic invariant for small scale turbulence in the presence of a mean flow. We have shown that the form of the matrix coefficient for the nonlocal coupling of the small scale fluctuations to the mean flow is crucial for the form of the adiabatic invariant. We have obtained adiabatic invariant $N_k = I_k k^2 \rho_s^2 (1 + k^2 \rho_s^2)$ for the drift wave turbulence described by the Hasegawa-Mima equation and isomorphic Charney-Obukhov equation for Rossby waves; and the invariant $N_k = I_k (1 + k^2 \rho_s^2)^2$ for the drift wave type turbulence in tokamaks such as TITG driven modes. [Note that the latter invariant reduces to the standard form [13, 14] for $k_\theta = const.$] The pseudo-action invariants appear in the wave kinetic equation and correspond to the quantities that are conserved as integrals over the small scale part of the spectrum alone. This specific conservation property makes them suitable as canonical Hamiltonian variables for small scale turbulence in the presence of the shear flow. The wave action invariants and the kinetic equation derived here can be used to investigate nonlinear dynamics of drift waves and zonal flow in a tokamak. The method used in our work can be applied to derive generalized invariants for other models including the Rossby type waves in geostrophic fluids [12].

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Table I: Generalized action invariants for different models

Model	Expression for the wave action
Drift waves in a sheared field, Eq.(12)	$I_k(1 + k^2\rho_s^2)^2$
Standard drift wave model, Eq.(17)	$I_k k^2\rho_s^2(1 + k^2\rho_s^2)$
2D Euler equation, Eq. (22)	$I_k k^4$